The AdS/CFT Correspondence

反德西特/共形场论对偶 (AdS/CFT 对应)

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A short introduction to the Anti-de-Sitter/Conformal Field Theory correspondence and related ideas.

本文对反德西特/共形场论对偶及相关思想做简要介绍。

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Introduction

引言

Quantum gravity in asymptotically anti-de-Sitter spacetimes has been conjectured to be equivalent to a conformal field theory on the boundary [1-3]. Here we introduce anti-de-Sitter space, some basic notions about conformal field theories and describe the general connection between the two. We then mention a particular example, the duality relating $AdS_5 \times S^5$ to $\mathcal{N} = 4$ supersymmetric Yang Mills theory. We describe some applications to black holes and conclude with some interesting connections between entanglement and spacetime geometry.

渐近反德西特时空下的量子引力被猜想等价于边界上的共形场论 [1-3]。本文我们将介绍反德西特空间、共形场论的若干基础概念，并阐述二者之间的一般联系。随后我们会给出一个具体例子，即联系 $AdS_5 \times S^5$ 与 $\mathcal{N} = 4$ 超对称杨-米尔斯理论的对偶性。我们还会介绍该对偶性在黑洞方面的若干应用，最后讨论纠缠与时空几何之间一些有趣的关联。

Gravity in Anti-de-Sitter Space

反德西特空间中的引力

The Geometry of Anti-de-Sitter Space

反德西特空间的几何学

Hyperbolic space is the simplest negatively curved space. If we make one of its coordinates time-like, then we obtain Anti-de-Sitter space, which is the simplest and most symmetric solution of Einstein's equations with a negative cosmological constant. Its metric can be written in a variety of coordinate systems. For example, in $d+1$ dimensions, we have

双曲空间是最简单的负曲率空间。如果我们将它的其中一个坐标改为类时坐标，就得到了反德西特空间，它是带负宇宙学常数的爱因斯坦方程最简单、对称性最高的解。它的度规可以在多种坐标系下写出。例如，在 $d+1$ 维中，我们有

$$ds^2 = R^2\left[\frac{-dx_0^2 + dx_1^2 + \cdots dx_{d-1}^2 + dz^2}{z^2}\right], \text{ Poincare coordinates} \tag{1}$$

$$ds^2 = R^2\left[-d\tau^2\cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_{d-1}^2\right], \text{ Global coordinates} \tag{2}$$

here R is a length scale that sets the radius of curvature of the spacetime. Via the Einstein equations, R can be expressed in terms of the Newton and cosmological constants. In order to see more clearly the symmetries of this space, it is convenient to view it as follows. We start from $R^{2,d}$, a flat space with two time directions with the metric

此处 R 是确定时空曲率半径的长度标度。通过爱因斯坦方程，R 可以用牛顿常数和宇宙学常数表示。为了更清楚地看出这个空间的对称性，我们可以方便地按如下方式构造: 我们从 $R^{2,d}$ 出发，这是一个带有两个时间方向的平直空间，其度规为

$$ds^2 = -dY_{-1}^2 - dY_0^2 + dY_1^2 + \cdots dY_d^2 \tag{3}$$

and we consider the surface

然后我们考虑如下曲面

$$-Y_{-1}^2 - Y_0^2 + Y_1^2 + \cdots Y_d^2 = -R^2 \tag{4}$$

Even though the starting spacetime had two time directions, the physical space (4) has only one time direction. To be more precise, the surface (4) has a non-contractible cycle, the angular direction in the first two directions. For the physical spacetime, we want to treat this direction as non-compact, which is the same as saying that the τ coordinate in (2) is non-compact. This presentation shows that the symmetry group is $\widetilde{SO}(2,d)$, with the tilde related to the standard non-compactness of the time τ .

尽管初始时空有两个时间方向，但式 (4) 的物理空间仅含一个时间方向。更准确地说，式 (4) 的曲面存在一个不可收缩闭链，对应前两个坐标的角方向。对于物理时空，我们要求该方向是非紧致的，这等价于说式 (2) 中的 τ 坐标是非紧致的。这种构造说明空间的对称群是 $\widetilde{SO}(2,d)$ ，波浪号和时间 τ 的标准非紧致性相关。

The global coordinates (2) cover all of AdS_{d+1} space, as their name indicates. The factor of $\cosh^2\rho$ can be viewed as a gravitational potential with a minimum at $\rho = 0$. In fact, $\rho = 0$ is a geodesic in this spacetime. Other geodesics oscillate around $\rho = 0$. All of these geodesics are related by the isometries. This means that an observer who is oscillating around $\rho = 0$ will feel that he/she is at rest. Due to the rapid rise of the gravitational potential, a massive particle cannot go all the way to $\rho = \infty$ if it departs from a finite value of ρ with a finite energy. On the other hand, light rays can go all the way to $\rho = \infty$, and they do this in a finite amount of time τ . So, if we set reflecting boundary conditions at infinity, they would bounce back to the center in a finite amount of time τ , which is actually $\tau = \pi$, see Fig. 1b. Even though the light rays go all the way to infinity, when they reach infinity their local wavelength becomes very large, as seen by an observer at a fixed ρ in that region. The Poincare coordinates (1) cover only a portion of the cylinder. In order to go between the different coordinate systems it is convenient to use the coordinates in (4) as an intermediate step by expressing (setting $R = 1$)

正如名称所示，整体坐标 (2) 可以覆盖整个 AdS_{d+1} 空间。$\cosh^2\rho$ 因子可以看作引力势，它在 $\rho = 0$ 处取得最小值。事实上，$\rho = 0$ 是该时空的一条测地线。其他测地线会围绕 $\rho = 0$ 振荡。所有这些测地线都可以通过等距变换联系起来，这意味着围绕 $\rho = 0$ 振荡的观测者会感受到自己处于静止状态。由于引力势上升极快，有质量粒子如果从有限 ρ 处、携带有限能量出发，不可能一直运动到 $\rho = \infty$ 。另一方面，光可以一直运动到 $\rho = \infty$ ，并且它会在有限的时间 τ 内到达。因此，如果我们在无穷远处设置反射边界条件，光会在有限时间 τ ——实际为 $\tau = \pi$ ——反弹回中心，参见图 1b。尽管光可以一直运动到无穷远，但固定在该区域 ρ 处的观测者会看到，光到达无穷远时其局域波长会变得非常大。庞加莱坐标 (1) 仅能覆盖圆柱体的一部分。为了在不同坐标系之间转换，以式 (4) 的坐标作为中间步骤会很方便，即通过 (令 $R = 1$)

$$Y_{-1} + Y_d = \frac{1}{z},\ Y_\mu = \frac{x_\mu}{z},\ \mu = 0, 1, \cdots d-1 \tag{5}$$

and solving for $Y_{-1} - Y_d$ using (4). Note that this expression makes it manifest that the coordinates (1) cover only the portion with $Y_{-1} + Y_d > 0$. Similarly, we have

再利用式 (4) 求解 $Y_{-1} - Y_d$ 。注意这个表达式清楚地表明坐标 (1) 仅覆盖满足 $Y_{-1} + Y_d > 0$ 的区域。类似地，我们有

$$Y_{-1} + iY_0 = \cosh\rho e^{i\tau},\ Y_i = n_i \sinh\rho,\ n_i n_i = 1 \tag{6}$$

From this we could find the relation between the two coordinate systems.

由此我们可以得到两个坐标系之间的关系。

Anti-de-Sitter space has a boundary, which is time-like. This is easily seen by considering the Penrose diagram of AdS . This can be easily obtained by dividing the metric in (2) by $\cosh^2\rho$. Then the geometry becomes that of (time) $\times B_d$ where B_d is a solid unit ball in R^d . The boundary is then (time) $\times S^{d-1}$, and it sits at $\rho \to \infty$.

反德西特空间存在一个类时边界。我们可以通过考察 AdS 的彭罗斯图很容易看出这一点: 将式 (2) 中的度规除以 $\cosh^2\rho$ 就很容易得到彭罗斯图。此时几何变成 (时间) $\times B_d$ ，其中 B_d 是 R^d 中的单位实心球。边界则为 (时间) $\times S^{d-1}$ ，位于 $\rho \to \infty$ 处。

The metric on the boundary is

边界上的度规为

$$ds^2 = \Omega^2 \left[-d\tau^2 + d\Omega_{d-1}^2 \right] \tag{7}$$

which is defined up to an overall function Ω of the rest of the coordinates. With the above procedure, we have $\Omega = 1$. However, we could have multiplied the scale factor by an arbitrary function to get a different metric. More precisely, the metric of the boundary is defined up to an overall scale, $g_{\mu\nu} \sim \Omega^2(x) g_{\mu\nu}$. In fact, if we multiplied (1) by z^2 and went to $z \to 0$, we would get the metric of $R^{1,d-1}$, which covers only a portion of the cylinder $R \times S^{d-1}$ and differs from it by an overall scale factor $\Omega^2(x)$.

它在整体坐标其余坐标的整体函数 Ω 范围内定义。通过上述步骤, 我们得到了 $\Omega = 1$。不过我们也可以给比例因子乘上任意函数, 得到不同的度量。更准确地说, 边界的度量在整体标度 $g_{\mu\nu} \sim \Omega^2(x) g_{\mu\nu}$ 范围内定义。事实上, 如果我们给 (1) 式乘上 z^2 , 再变换到 $z \to 0$, 就会得到 $R^{1,d-1}$ 的度量, 这个度量仅覆盖圆柱 $R \times S^{d-1}$ 的一部分, 且和原度量相差一个整体比例因子 $\Omega^2(x)$ 。

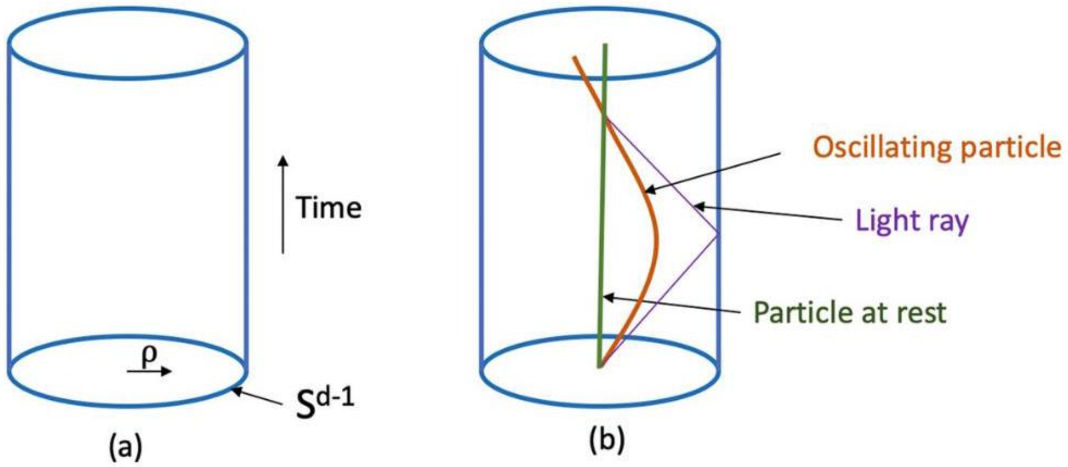


Fig. 1 (a) Anti-de-Sitter Penrose diagram. The boundary is a cylinder, the vertical direction is time, and spatial slice is a $d-1$ dimensional sphere. The interior of the cylinder is AdS . (b) Trajectories of massive particles and of a light ray that bounces from the boundary

图 1 (a) 反德西特空间彭罗斯图。边界是一个圆柱, 竖直方向为时间, 空间切片是一个 $d-1$ 维球面。圆柱内部是 AdS 。(b) 有质量粒子和从边界反射的光线的轨迹

In the coordinates (4), we can think of the boundary as the region where $Y^M \to \infty$. Rescaling them by an overall infinite factor, we define the finite coordinates X^M , which obey

在坐标 (4) 中, 我们可以将边界看作满足 $Y^M \to \infty$ 的区域。将它们乘上整体无穷因子做重标度后, 我们定义有限坐标 X^M , 满足

$$-X_{-1}^2 - X_0^2 + X_1^2 + \cdots X_d^2 = 0, \; X^M \sim \lambda X^M \tag{8}$$

where the second relation arises from the fact that the rescaling factor is arbitrary, and it means that the coordinates X^M are projective coordinates. Furthermore, on this surface the group $\widetilde{SO}(2,d)$ acts as conformal transformations. Let us discuss this last point in more detail. We can define a metric on (8) as

其中第二个关系来源于重标度因子任意性，它说明坐标 X^M 是射影坐标。此外，群 $\widetilde{SO}(2,d)$ 在该曲面上作为共形变换作用。下面我们更详细地讨论这一点。我们可以在 (8) 式上定义度量如下:

$$ds^2 = \frac{dX.dX}{\left(X_{-1} + X_d\right)^2},\ X.X = 0,\ X \sim \lambda X \tag{9}$$

where we chose one quadratic function of the coordinates so as to make the metric invariant under the rescaling in (8). This metric, (9), reduces to the standard flat space metric, $dx_\mu dx^\mu$, after defining

此处我们选取了坐标的一个二次函数，使得度量在 (8) 式的重标度下不变。这个度量，即 (9) 式，在定义 $dx_\mu dx^\mu$ 后约化为标准平直空间度量，

$$x^\mu = \frac{X^\mu}{(X_{-1} + X_d)},\ \mu = 0, \cdots d-1 \tag{10}$$

We now see that an $\widetilde{SO}(2,d)$ would change the metric (9) only by an overall factor, since the overall factor is not invariant. Transformations that leave the metric invariant up to an overall factor are called "conformal" transformations. This group of transformations include the ordinary Poincare transformations (translations, boosts, and rotations) plus a scaling transformation $x^\mu \to \sigma x^\mu$ and special conformal transformations, which infinitesimally act like $\delta x^\mu = b^\mu x^2 - 2x^\mu (b.x)$. These latter ones can be obtained by conjugating the translations with the inversion $x^\mu \to \frac{x^\mu}{x^2}$. Note that if we replaced the overall factor in (9) by $X_{-1}^2 + X_0^2$, we would get the metric of $R \times S^{d-1}$ as in (7). In conclusion, the $\widetilde{SO}(2,d)$ isometries of AdS act on its boundary as the group of conformal transformations in a d -dimensional spacetime.

我们现在可以看到，$\widetilde{SO}(2,d)$ 只会改变度量 (9) 的一个整体因子，因此整体因子并不不变。仅相差一个整体因子、仍保持度量不变的变换被称为「共形」变换。这个变换群包含普通庞加莱变换 (平移、快度 boost 和转动) 加上标度变换 $x^\mu \to \sigma x^\mu$ 和特殊共形变换，特殊共形变换的无穷小作用形式为 $\delta x^\mu = b^\mu x^2 - 2x^\mu (b.x)$ 。后者可以通过平移与反演变换 $x^\mu \to \frac{x^\mu}{x^2}$ 共轭得到。注意如果我们将 (9) 中的整体因子替换为 $X_{-1}^2 + X_0^2$ ，我们会得到和 (7) 中一致的 $R \times S^{d-1}$ 的度量。综上，AdS 的 $\widetilde{SO}(2,d)$ 个等距变换在其边界上作用为 d 维时空上的共形变换群。

It is often useful to consider the Euclidean continuation of Anti-de-Sitter space, sometimes called Euclidean AdS or, more traditionally, hyperbolic space. This is just obtained by switching the sign of the dx_0^2 in (1), or $d\tau^2$ in (2), or Y_0^2 in (4).

考虑反德西特空间的欧几里得延拓通常很有用，它有时被称为欧几里得 AdS，更传统的名称是双曲空间。它只需要改变 (1) 中 dx_0^2 的符号，或 (2) 中 $d\tau^2$ 的符号，或 (4) 中 Y_0^2 的符号即可得到。

Quantum Fields in AdS Space

AdS 空间中的量子场

We can consider quantum fields in Anti-de-Sitter space. Since AdS has an everywhere time-like Killing vector, associated to ∂_τ , we can define a vacuum by picking the usual positive frequency condition. We can consider the Hamiltonian that generates τ translations, and call it H_τ . As we remarked above, the motion of classical particles is oscillatory around $\rho = 0$ and, for this reason, we expect that energies should be quantized. In order to study the spectrum of a free field, it is very convenient to use the symmetries. Defining $Y_\pm = Y_{-1} \pm Y_0$, the $M_{\pm,i}$ generators of the group act as raising or lowering operators. In addition, the Casimir, which is quadratic in the generators, is a second-order differential operator, which corresponds to the wave equation. So the mass of the field determines the eigenvalue of the Casimir. Together with the intrinsic spin of the field, this defines the representation of the $\widetilde{SO}(2, d-1)$ group we should consider.

我们可以研究反德西特空间中的量子场。由于 AdS 处处存在与 ∂_τ 相关的类时基灵矢量，我们可以通过选取常规正频率条件定义真空。我们可以考虑生成 τ 平移的哈密顿量，将其记为 H_τ 。正如我们前文所述，经典粒子会围绕 $\rho = 0$ 做振荡运动，因此我们预期能量应当是量子化的。研究自由场的能谱时，利用对称性非常方便。定义 $Y_\pm = Y_{-1} \pm Y_0$ 后，群的 $M_{\pm,i}$ 生成元可作为升算符或降算符使用。此外，生成元的二次型卡西米尔算符是一个二阶微分算符，对应波动方程。因此场的质量决定了卡西米尔算符的本征值。结合场的内禀自旋，这就确定了我们需要研究的 $\widetilde{SO}(2, d-1)$ 群的表示。

For the simplest case of the scalar field of mass m , we can find the lowest-energy state annihilated by the corresponding lowering operators. This state has energy [2, 3]

对于质量为 m 的标量场这一最简单情形，我们可以找到被对应降算符湮灭的最低能量态。该态的能量为 [2, 3]

$$\Delta = H_\tau = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 R^2} \tag{11}$$

We have defined the eigenvalue of H_τ as Δ , which is the conventional notation for scaling dimension in the conformal group. The wavefunction of this state is simply

我们将 H_τ 的本征值定义为 Δ ，这是共形群标度维度的常规记号。该态的波函数简单表示为

$$\phi \sim \frac{e^{-i\Delta\tau}}{(\cosh\rho)^\Delta} \tag{12}$$

All other states have energies $H_\tau = \Delta + n$, and their wavefunctions can be obtained by acting with the isometries. So we see that we get a spectrum similar to that of a harmonic oscillator. This is the spectrum of single particle states of the quantum field. In the full quantum field theory, we can also have multiparticle states with energies, which are the sum of the energies of the single particle states $H_\tau = \sum_i \Delta_i + n_i$. This is the result if the field theory is free. If there are interactions, then the problem of computing the spectrum is more complicated. However, if the interactions are weak, this will give the leading order answer.

所有其他态的能量为 $H_\tau = \Delta + n$ ，它们的波函数可以通过等距变换作用得到。因此我们可以看到，此处能谱与简谐振子的能谱类似。这就是量子场单粒子态的能谱。在完整量子场论中，我们还可以得到能量为各单粒子态能量之和 $H_\tau = \sum_i \Delta_i + n_i$ 的多粒子态。这是场论为自由场时的结果。如果存在相互作用，计算能谱会复杂得多。但如果相互作用很弱，上述结果就是领头阶的答案。

Notice that for large $m \gg 1/R$ the energy (11) is approximately $H_\tau \sim mR$. This is simply the usual relativistic energy mc^2 , with $c = 1$ and accounting for the overall factor of R in the metric 2 .

注意，对于大 $m \gg 1/R$ ，(11) 式给出的能量近似为 $H_\tau \sim mR$ 。这就是常规相对论能量 mc^2 ，其中 $c = 1$ ，并且考虑了度规 (2) 中的整体因子 R 。

Let us comment on a small subtlety. In writing the energies (11), we have implicitly put a boundary condition at infinity. At infinity, the solutions of the wave equation go like $\rho^{-\Delta}, \rho^{\Delta-d}$. When $m^2 \geq 0$ only one of them decays. It turns out that it is possible to have particles with $m^2R^2 < 0$ as long, and this is not too negative $m^2R^2 > -\frac{d^2}{4}$ [4]. In the range $1 - d^2/4 > m^2R^2 > -d^2/4$ it is possible to choose the other sign in front of the square root (11) [5], which amounts to a different boundary condition at infinity. This then gives us the full range of discrete unitary representations of the conformal group $\Delta > \frac{d-2}{2}$. The representation with $\Delta = \frac{d-2}{2}$ is also unitary, but it corresponds to a free massless field purely at the boundary [6].

我们来讨论一个小细节。在写出能量 (11) 时，我们已经隐含在无穷远施加了边界条件。在无穷远，波动方程的解行为近似为 $\rho^{-\Delta}, \rho^{\Delta-d}$ 。当 $m^2 \geq 0$ 时，只有其中一个解是衰减的。研究表明，只要 $m^2R^2 > -\frac{d^2}{4}$ 不是太小负，就可以存在质量为 $m^2R^2 < 0$ 的粒子 [4]。在范围 $1-d^2/4 > m^2R^2 > -d^2/4$ 内，我们可以在 (11) 式的根号前选择另一个符号 [5]，这对应于无穷远处的另一种边界条件。由此我们可以得到共形群全部的离散幺正表示 $\Delta > \frac{d-2}{2}$ 。$\Delta = \frac{d-2}{2}$ 对应的表示也是幺正的，但它对应于一个完全在边界上的自由无质量场 [6]。

Gravity in Asymptotically Anti-de-Sitter Spacetimes

渐近反德西特时空的引力

We now consider anti de Sitter space in a theory of gravity. If we simply consider small fluctuations around anti-de-sitter space, the graviton is just one more field, and it gives rise to a particular series of states. The lowest-energy state has spin two and dimension $\Delta = d$.

我们现在在引力理论中研究反德西特空间。如果我们仅考虑反德西特空间的小涨落，引力子只是额外的一个场，它会产生一系列特殊的态。最低能量态的自旋为 2，维度是 $\Delta = d$ 。

In a quantum theory of gravity, we are supposed to sum over all spacetimes. In other words, we expect that quantum fluctuations can change the geometry and the topology of spacetime. However, we expect that we can fix the asymptotic form of the spacetime. For example, we can fix it to be asymptotically anti-de-Sitter. We can impose this condition when we have a negative cosmological constant. In other words, the metric should have the form

在量子引力理论中，我们需要对所有时空求和。换句话说，我们认为量子涨落可以改变时空的几何与拓扑。但是，我们可以固定时空的渐近形式。例如，我们可以将其固定为渐近反德西特。当我们拥有负宇宙学常数时就可以施加这个条件。也就是说，度规应取如下形式

$$ds^2 \sim e^{2\rho}\left[-d\tau^2 + d\Omega_{d-1}^2\right] + d\rho^2 + \cdots, \ \rho \gg 1 \tag{13}$$

at large ρ . The dots denote terms that are subleading at large ρ . At large ρ , there can still be quantum fluctuations of the metric at distances of order the AdS radius. However, in (13) we are keeping the angular coordinate differences fixed as we go to large ρ , which means that we looking at extremely long proper distances. Quantum fluctuations are small at such long distances, so it makes sense to fix the geometry. Notice that this also amounts to a restriction on the energies of states we consider. We consider states with finite energy E as we take the $\rho \to \infty$ limit. We do not consider states that have infinite energy!. This might seem a silly comment; however, if AdS is produced through a bubble nucleation event, then the state that is produced typically has infinite energy compared to the AdS vacuum [7]. In other words, we are talking about AdS spaces, which are completely cold asymptotically, the large ρ region is locally in the vacuum.

在大 ρ 处成立。省略号代表大 ρ 下的次领头项。在大 ρ 处，尺度为 AdS 半径的距离上仍然可以存在度规的量子涨落。但是，在 (13) 式中，当我们趋近大 ρ 时，我们保持角坐标差固定，这意味着我们观测的是极长的固有距离。量子涨落在这种长距离下很小，因此固定几何是合理的。注意，这也相当于对我们考虑的态的能量做出了限制。当我们取 $\rho \to \infty$ 极限时，我们研究的是能量有限 E 的态。我们不考虑能量无限的态！这看起来像是一句多余的话；但如果反德西特空间是通过泡核形成过程产生的，那么产生的态相较于 AdS 真空通常具有无限能量 [7]。换句话说，我们讨论的是渐近完全低温的 AdS 空间，大 ρ 区域局域处于真空。

These boundary conditions only fix the asymptotic structure of spacetime, but not the spacetime in the interior. We can have black holes or geometries that are connected to other asymptotic regions (more on this later).

这些边界条件仅固定了时空的渐近结构，并不固定内部时空。内部可以存在黑洞，或是连接其他渐近区域的几何 (我们后续会展开讨论)。

This asymptotically AdS gravity theory might be ill-defined if the vacuum can decay into other, lower cosmological constant spacetimes. In other words, to have a well-defined problem, we need to ensure that the asymptotic vacuum is absolutely stable. This can be ensured if the theory is supersymmetric. We do not currently know whether it is possible to achieve this without supersymmetry in theories that are approximated by Einstein gravity (It has been speculated that it is impossible in [8].). In fact, it is interesting that AdS space can preserve some supersymmetries, so that the symmetry group is enhanced to a ”superconformal” group.

如果真空可以衰变为其他宇宙学常数更低的时空，这个渐近反德西特引力理论可能是不适定的。也就是说，要得到一个适定的问题，我们需要确保渐近真空是绝对稳定的。如果理论是超对称的，就可以满足这个要求。目前我们尚不清楚，在可以用爱因斯坦引力近似的理论中，能否在没有超对称的情况下实现这一点 (文献 [8] 推测这不可能)。事实上，值得注意的是反德西特空间可以保留部分超对称性，因此对称群会增强为「超共形」群。

Validity of the Gravity Approximation

引力近似的有效性

In the previous subsection, we discussed the asymptotically AdS gravity problem as if we knew what the full theory of quantum gravity is. However, we presently do not have any formulation of quantum gravity for general spacetimes. However, we have some approximate ways to discuss such theories. The first approach is that of gravity as an effective field theory. This approach is applicable for relatively low energies, or when the curvatures are sufficiently small compared to the Planck scale. In AdS, this amounts to the condition that

> 在上一小节中，我们讨论了渐近反德西特引力问题，讨论过程预设我们已经知晓量子引力的完整理论。然而，目前我们尚不存在可适用于一般时空的量子引力表述，不过我们有一些近似方法可以讨论这类理论。第一种方法是将引力视为有效场论。该方法适用于能量相对较低，或是曲率远小于普朗克尺度的情况。在反德西特空间中，这等价于满足以下条件

$$c_{\text{grav}} = \frac{R^{d-1}}{G_N} \gg 1 \tag{14}$$

Notice that this quantity is dimensionless. $1/c_{\text{grav}}$ sets the strength of gravity at scales comparable to the radius of AdS. This semiclassical approach to gravity will be well defined as long a c_{grav} is large, and $1/c_{\text{grav}}$ is the effective expansion parameter. In this expansion, we encounter unknown counterterms at higher orders, and so the theory is not precisely defined. Nevertheless, we can still do many approximate computations in the regime in (14), but we do not know whether there is a fully defined theory that has the given theory as its low-energy expansion.

> 请注意该量是无量纲的。$1/c_{\text{grav}}$ 决定了引力在与反德西特半径相当的尺度上的强度。只要 c_{grav} 足够大，这种引力的半经典方法就是良定义的，且 $1/c_{\text{grav}}$ 是有效展开参数。在该展开中，高阶项会出现未知抵消项，因此该理论无法被精确定义。尽管如此，我们仍然可以在条件 (14) 的适用范围内进行许多近似计算，但我们无法确定是否存在一个定义完整的理论，将给定理论作为自身的低能展开。

A theory that is better defined is string theory [9]. In this case, the complete perturbative expansion in powers of the string coupling is well defined. It is believed that this expansion defines a unique theory non-perturbatively. In string theory, there is a new length scale, the string length l_s, which is larger than the Planck length for weak string coupling. In string theory, the graviton is a small string of size l_s, so the gravity approximation breaks down when we go to distances similar to l_s. So if we want to apply the usual gravity approximation we should require that

> 弦论 [9] 是一个定义更完善的理论。在弦论中，按弦耦合幂次展开的完整微扰展开是良定义的，人们普遍认为该展开非微扰地定义了一个唯一的理论。弦论中存在一个新的长度尺度，即弦长 l_s，在弦耦合较弱时，弦长大于普朗克长度。弦论中，引力子是大小为 l_s 的小弦，因此当我们探究尺度与 l_s 相近的距离时，引力近似就会失效。所以如果我们想要应用常规引力近似，就需要满足条件

$$\frac{R}{l_s} \gg 1 \tag{15}$$

If this condition is not obeyed, the full string theory is still well defined, but it might be harder to analyze since it does not reduce to Einstein gravity. We should also mention that in string theory we have $c_{\text{grav}} \propto 1/g_s^2$ where g_s is the string coupling.

> 如果不满足该条件，完整弦论仍然是良定义的，但分析难度会提升，因为它无法约化为爱因斯坦引力。我们还需要指出，弦论中存在 $c_{\text{grav}} \propto 1/g_s^2$，其中 g_s 是弦耦合。

Some Interesting Gravity Computations

若干有趣的引力计算

We will mention here some interesting computation that can be done in any theory of gravity in AdS . The simplest computations involve the Euclidean case. One can consider the Euclidean AdS spacetime written as

我们在此介绍一些可在 AdS 中任意引力理论里完成的有趣计算。最简单的计算涉及欧几里得情形。我们可以将欧几里得 AdS 时空写为

$$ds^2 = R^2 \left[d\sigma^2 + \sinh^2\sigma d\Omega_d^2\right] \tag{16}$$

where the boundary is a Euclidean sphere S^d . We can compute the gravitational action on this geometry by integrating

其边界是一个欧几里得球面 S^d 。我们可以通过积分计算该几何上的引力作用量

$$-I = \frac{1}{16\pi G_N}\left[\int_{H_{d+1}} \sqrt{g}\,(R-2\Lambda) + 2\int_{\partial H_{d+1}} \sqrt{h}K\right],\ \Lambda = -\frac{d\,(d-1)}{2R^2}$$

(17)

where h is the metric restricted to the boundary. It turns out that this is actually infinite basically because the bulk term is proportional to the volume and the volume is infinite. Before getting disappointed, it is useful to understand the origin and the nature of this infinity. For this purpose, it is better to consider a slight generalization of the problem. Imagine that we fix a boundary metric of the form

其中 h 是限制在边界上的度规。事实上该结果是发散的，这本质上是因为体元项正比于体积，而体积是无穷大。在失望之余，理解这个无穷大的起源与性质十分有益。为此，我们最好对问题做一个轻微推广: 假设我们固定如下形式的边界度规

$$g_{\mu v}|_{bdy} = h_{\mu v} = \frac{1}{\varepsilon^2}\hat{h}_{\mu v} \tag{18}$$

where μ, ν are indices along the boundary. We will be interested in taking $\varepsilon \to 0$ keeping $\hat{h}_{\mu\nu}$ finite. Then, for any finite ε , we can ”fill in” the space with a metric obeying Einstein equations with a cosmological constant, which will give us an Einstein space, with $R_{ab} \propto -g_{ab}$, where a, b are bulk indices (Fig. 2). We then compute the action using (17), which gives us a finite answer. The divergence then appears as we take $\varepsilon \to 0$. In (16) $\hat{h}_{\mu\nu}$ is the metric of a unit sphere. However, to shed some light on the divergence, it is better to keep it general. We then find that the most divergent term goes as

其中 μ, ν 是沿边界的指标。我们将重点考虑取 $\varepsilon \to 0$ 、同时保持 $\hat{h}_{\mu\nu}$ 有限的情况。那么对任意有限的 ε，我们可以用满足带宇宙常数爱因斯坦方程的度规“填充”该空间，得到一个爱因斯坦空间，满足 $R_{ab} \propto -g_{ab}$ ，其中 a, b 是体指标 (图 2)。随后我们利用 (17) 计算作用量，就能得到有限结果。发散会在我们取 $\varepsilon \to 0$ 极限时出现。在 (16) 中，$\hat{h}_{\mu\nu}$ 是单位球面的度规。不过为了弄清楚发散的性质，我们最好保持其一般性。我们会发现发散最强的项形式如下

$$-I \sim \frac{(d-1)}{8\pi G_N}\frac{1}{R}\int\sqrt{h} = \frac{(d-1)}{8\pi G_N}\frac{1}{R}\frac{1}{\varepsilon^d}\int\sqrt{\hat{h}} + \cdots \tag{19}$$

We see that this is a very simple term. It is given purely in terms of the boundary metric $\hat{h}$, and we do not need to know the details of the solution that "fills it in," which is the complicated part of the problem. This expression is a single integral and can be viewed as a purely local contribution since it depends locally on the boundary data given by h_{ij} . When we evaluate the action, we can then subtract this local divergence and get something less divergent. It turns out that there are subleading divergencies, but they are all of the local form, except that they involve higher powers of the curvature tensors [11,13]. For example, the next correction involves $\int\sqrt{h}R_h$ where R_h is the Ricci scalar for the metric $h_{\mu\nu}$. We can view these contributions as "local counterterms" that we need to add to make the final result finite. These do not depend on the bulk solution. It requires some computation to show that after subtracting these local terms, the resulting answer is finite.

可以看到这是一个非常简单的项，它完全由边界度规 $\hat{h}$ 给出，我们不需要知道“填充”这个过程对应解的细节——那才是问题的复杂部分。这个表达式是单个积分，可以看作纯局域贡献，因为它局域依赖于 h_{ij} 给出的边界数据。计算作用量时，我们可以减去这个局域发散，得到发散程度更低的结果。事实上还存在次领头发散，但它们也全都是局域形式，只不过会涉及曲率张量的更高次幂[11,13]。例如，下一个修正项包含 $\int\sqrt{h}R_h$ ，其中 R_h 是度规 $h_{\mu\nu}$ 的里奇标量。我们可以将这些贡献视为“局域抵消项”，添加它们就能让最终结果有限。这些抵消项不依赖于体解，通过计算可以证明，减去这些局域项后，得到的结果是有限的。

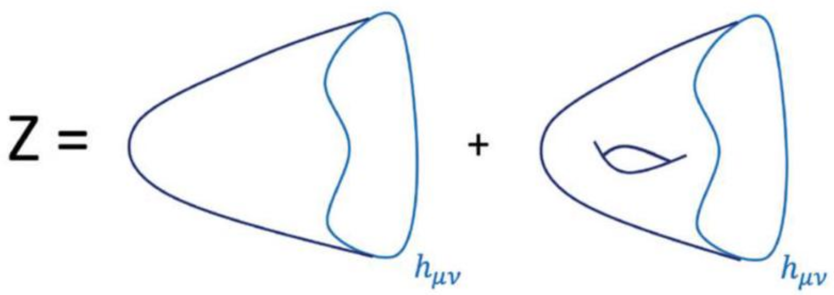


Fig. 2 The gravity partition function with given boundary condition can be viewed as a sum over geometries, which obey the asymptotically AdS boundary conditions. Classically the first geometry is found by solving Einstein's equations with asymptotically AdS boundary conditions. In practice, we only know how to perform this sum in very special two-dimensional theories of gravity [16, 17, 17 − 19]

图 2 给定边界条件的引力配分函数可以看作对满足渐近 AdS 边界条件的几何的求和。经典层面上，第一个几何就是通过求解渐近 AdS 边界条件下的爱因斯坦方程得到的。实践中，我们仅知道如何在非常特殊的二维引力理论 [16, 17, 17 − 19] 中完成这个求和

These subtractions are simplest in the case of odd d . After we subtract all these terms, the answer does not depend on the overall scale of the metric h . For the sphere we find that integral for the vacuum hyperbolic space gives [12]

当 d 为奇数时，这些减法操作最简单。减去所有这些项后，结果不依赖于度规 h 的整体标度。对于球面，我们可以得到真空双曲空间的积分结果为 [12]

$$\log Z \sim -I = -\frac{\pi R^2}{2G_N}, \text{ for } d = 3. \tag{20}$$

We can think of this as the leading approximation to a gravity partition function, Z , which is formally the sum over all geometries whose boundary is a sphere. The right-hand side is the leading classical answer, the next term comes from the determinants of the quantum fields around hyperbolic space.

我们可以将其看作引力配分函数 Z 的领头阶近似，该配分函数形式上就是对所有边界为球面的几何求和。右侧是领头阶经典结果，下一项来自双曲空间背景下量子场的行列式。

For even d there is a logarithmic term in ε and the finite answer, after subtractions, actually depends on the overall scale factor of h . This can be viewed as a conformal anomaly and the variation of the action is local and expressible in terms of the curvature tensor [3, 10] .

当 d 为偶数时，结果中会出现一个关于 ε 的对数项，减法后得到的有限结果实际上依赖于 h 的整体标度因子。这可以视为共形反常，作用量的变分是局域的，可以通过曲率张量 [3, 10] 表示。

Another interesting computation is the following (Fig. 3). We consider a boundary, which is $S^1 \times S^{d-1}$. One possible Euclidean geometry is (2) with $\tau \to i\tau_E$ and $\tau_E \sim \tau_E + \beta$. The gravity action is then proportional to β , and the proportionality constant can be viewed as a vacuum energy or Casimir energy on the sphere. A more interesting geometry is that of the Euclidean Schwarzschild AdS black hole

另一项有趣的计算如下 (图 3)。我们考虑一个边界，即 $S^1 \times S^{d-1}$ 。存在一个可能的欧几里得几何，它满足式 (2)，且包含 $\tau \to i\tau_E$ 和 $\tau_E \sim \tau_E + \beta$ 。引力作用量正比于 β ，比例常数可视为球面上的真空能或卡西米尔能。更有趣的几何是欧几里得史瓦西 AdS 黑洞的几何

$$ds^2 = R^2\left[f d\tau_E^2 + \frac{dr^2}{f} + r^2 d\Omega_D^2\right], \; f = 1 + r^2 - \frac{\mu}{r^{d-2}} \tag{21}$$

where μ is adjusted so that when $f = 0$ the τ_E circle shrinks smoothly. This gives μ as a function of β . There is a real μ solution only for β sufficiently small. Subtracting the divergencies as above, we can evaluate the action. In this case, the answer is a nontrivial function of β , and for that reason, we get a nonzero entropy, which is given by the area of the horizon $S = A/4G_N$.

其中调整 μ 后，当 $f = 0$ 时，τ_E 圆会平滑收缩。由此可得 μ 是 β 的函数。仅当 β 足够小时，才存在 μ 的实解。和上文一样减去发散项后，我们可以算出作用量。这种情况下，结果是 β 的非平凡函数，因此我们得到非零熵，由视界面积 $S = A/4G_N$ 给出。

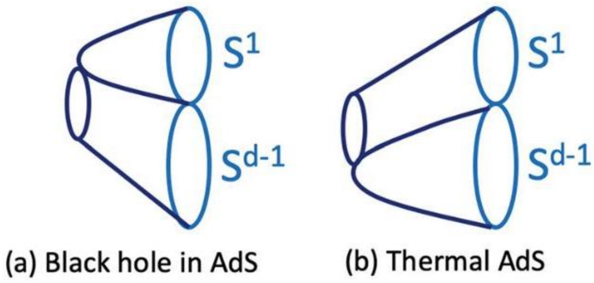


Fig. 3 Two ways to fill in the same boundary conditions, which are those appropriate for the thermal partition function on the sphere, namely $S^1 \times S^{d-1}$. (a) The Euclidean AdS Schwarzschild black hole where the S^1 is contractible in the full geometry. The circle shrinks to zero at the horizon. (b) Global Euclidean AdS with Euclidean time identified. Here the S^{d-1} shrinks to zero in a smooth way at the origin and the circle never shrinks to zero size

图 3 填充相同边界条件的两种方式，这些边界条件适用于球面上的热配分函数，即 $S^1 \times S^{d-1}$ 。(a) 欧几里得 AdS 史瓦西黑洞，其中 S^1 在完整几何中可收缩。该圆在视界处收缩为零。(b) 整体欧几里得 AdS ，欧几里得时间被等同。这里 S^{d-1} 在原点处平滑收缩为零，而原圆永远不会收缩到零尺寸

We see that for sufficiently small β there are at least two candidate solutions. Which one should we pick? In principle, we should sum over both of them, but the one with the lowest action dominates. The difference between the two actions leads to [14, 15]

我们可以看到，当 β 足够小时，至少存在两个候选解。我们该选择哪一个？原则上我们应当对二者都求和，但作用量最低的解占主导。两个作用量的差给出 [14, 15]

$$\log Z_{\mathrm{BH}} - \log Z_{\mathrm{ThAdS}} = -I_{\mathrm{BH}} + I_{\mathrm{ThAdS}} = \frac{\mathrm{Vol}\left(S^{d-1}\right) r_+^{d-1}\left(1 - r_+^2\right)}{4G_N\left(d r_+^2 + d - 2\right)} \tag{22}$$

where r_+ is the largest root of $f = 0$ in (21). We see that for $r_+ > 1$, the black hole solution dominates. This happens when the temperature is sufficiently large. This is a first-order phase transition, the entropy is discontinuous because it is zero for the first solution but nonzero for the black hole solution [14].

其中 r_+ 是式 (21) 中 $f = 0$ 的最大根。我们可知，对于 $r_+ > 1$ ，黑洞解占主导。这种情况发生在温度足够高时。这是一级相变，熵不连续，因为第一个解的熵为零，而黑洞解的熵非零 [14]。

Conformal Field Theories

共形场论

A conformal field theory is a special kind of quantum field theory. It is a quantum field theory that has no dimensionful parameters and has a scaling symmetry. Scaling symmetry generally leads to a conformal symmetry. Conformal transformations are coordinate transformations that preserve the metric up to an overall scale factor and were described around equation (9) (The case of $d = 2$ is special, and we have an infinite-dimensional group called the ”Virasoro” group of conformal symmetries. Only an $\widetilde{SO}(2,2)$ subgroup leaves the vacuum invariant, the others create some excitations.).

共形场论是一类特殊的量子场论，它是不含量纲参数、具有标度对称性的量子场论。标度对称性通常会导出共形对称性。共形变换是保度量差一个整体尺度因子的坐标变换，我们已经在方程 (9) 附近做了描述 ($d = 2$ 的情况比较特殊，我们会得到一个称为“Virasoro 群”的无限维共形对称群，仅存在一个 $\widetilde{SO}(2,2)$ 子群保持真空不变，其余子群都会产生激发)。

For theories that are conformal invariant, the trace of the stress tensor vanishes, $T_\mu^\mu = 0$. This implies that the theory is also Weyl invariant, namely we can change the scale factor of the metric of the space where the field theory is defined, $h_{\mu\nu} \to \Omega^2 h_{\mu\nu}$ for any function $\Omega(x)$. To be precise, for even d , this symmetry is anomalous, meaning that the partition function changes by a c-number, which involves the curvatures. This can be also expressed as saying that the trace of the stress tensor is proportional to terms that involve the background curvatures. It is important that this anomaly just involves the background fields and not other operators of the theory. So the dependence of the partition function on the scale factor of the metric is completely determined.

对于共形不变的理论，能动量张量的迹为零， $T_\mu^\mu = 0$ 。这意味着该理论也具有外尔不变性: 即我们可以改变场论定义所在空间的度量尺度因子， $h_{\mu\nu} \to \Omega^2 h_{\mu\nu}$ 对任意函数 $\Omega(x)$ 都成立。准确来说，即使是对 d ，该对称性也存在反常，即配分函数会多出一个 c 数项，该项与曲率有关，也可以表述为能动量张量的迹正比于包含背景曲率的项。重要的是，这种反常仅涉及背景场，不涉及理论的其他算符，因此配分函数对度量尺度因子的依赖是完全确定的。

There are many conformal field theories, and they can be defined in various ways. Simple examples are a free massless scalar, a free massless fermion field, a $d = 4$ Maxwell field. All these examples are free, but there are also examples of interacting CFTs, which will be more interesting for our purposes.

共形场论种类丰富，可以通过多种方式定义。简单的例子包括无质量自由标量场、无质量自由费米子场、$d = 4$ 麦克斯韦场。所有这些例子都是自由理论，但也存在相互作用共形场论的例子，对于我们的讨论而言，这类共形场论更有趣。

A basic property of the quantum field theory is a measure of the ”number of fields” it contains. This is a tricky thing to measure when we have an interacting theory. One way to define it is via the two-point function of the stress tensor operator, which has the form [20]

量子场论的一个基本性质是度量它所包含的“场的数目”，在相互作用理论中这个量并不容易定义。一种定义方式是通过能动量张量算符的两点关联函数，其形式为 [20]

$$\langle T_{\mu\upsilon}(x) T_{\mu'\upsilon'} \rangle = \frac{c_T}{x^{2d}} I_{\mu\upsilon,\mu'\upsilon'} \tag{23}$$

where I is a definite tensor involving the indices and x^μ with weight zero under scalings of x [20]. Up to the overall constant, this form of the two-point function is fixed by conformal symmetry and the conservation of the stress tensor. The coefficient c_T is a measure of the number of fields of the CFT (There are other measures that are better from the point of view of the renormalization group flow [21,22], but this one is useful for our purposes.).

其中 I 是一个确定的含指标张量，且在 x 的标度变换下 x^{μ} 权重为零 [20]。除整体常数外，两点关联函数的这个形式由共形对称性和能动量张量守恒完全确定。系数 c_T 就是对共形场论场数目的度量 (从重整化群流的角度来看，存在其他更合适的度量 [21,22]，但这个定义对我们的讨论已经足够)。

One way to compute this correlator is the following. We can consider the CFT on an arbitrary background metric $h_{\mu\nu}$ and compute the partition function on that metric. This partition function will have UV divergences, which involve integrals of $\int \sqrt{h}$ and $\int \sqrt{h} R_h$ and so on. After these divergences are extracted, the renormalized partition function, $Z[h]$, is a Weyl invariant function of h up to the conformal anomaly. We can then take functional derivatives to extract correlators.

计算该关联函数的一种方法如下: 我们可以将共形场论放在任意背景度量 $h_{\mu\nu}$ 上，计算该度量下的配分函数。这个配分函数会存在紫外发散，发散项包含 $\int \sqrt{h}$ 、$\int \sqrt{h} R_h$ 等的积分。提取出这些发散项后，重整化配分函数 $Z[h]$ 作为 h 的函数，除共形反常外满足外尔不变性，之后我们可以通过泛函导数提取出关联函数。

$$\langle T_{\mu_1\nu_1}(x_1)\, T_{\mu_2\nu_2}(x_2) \cdots T_{\mu_n\nu_n}(x_n)\rangle =$$

$$= \frac{\delta}{\delta h_{\mu_1\nu_1}(x_1)} \frac{\delta}{\delta h_{\mu_2\nu_2}(x_2)} \cdots \frac{\delta}{\delta h_{\mu_n\nu_n}(x_n)} \frac{Z[h]}{Z[\eta]}\bigg|_{h_{\mu\nu}\to\eta_{\mu\nu}} \tag{24}$$

where we set the background metric to the flat metric after taking the derivatives.

取完导数后我们再将背景度量设定为平直度量。

The conformal symmetry constrains these correlators. The two- and three-point functions are determined up to a few constants [20], but starting from the four-point functions we can start having arbitrary functions of conformal cross-ratios (combinations of coordinates that are conformal invariant). These correlators contain more information about the details of the CFT that we are considering.

共形对称性会对这些关联函数给出约束: 两点和三点关联函数除少数几个常数外被完全确定 [20]，但从四点关联函数开始，关联函数可以依赖共形交叉比 (坐标的共形不变组合) 的任意函数，这些关联函数包含了我们所研究的共形场论更多细节信息。

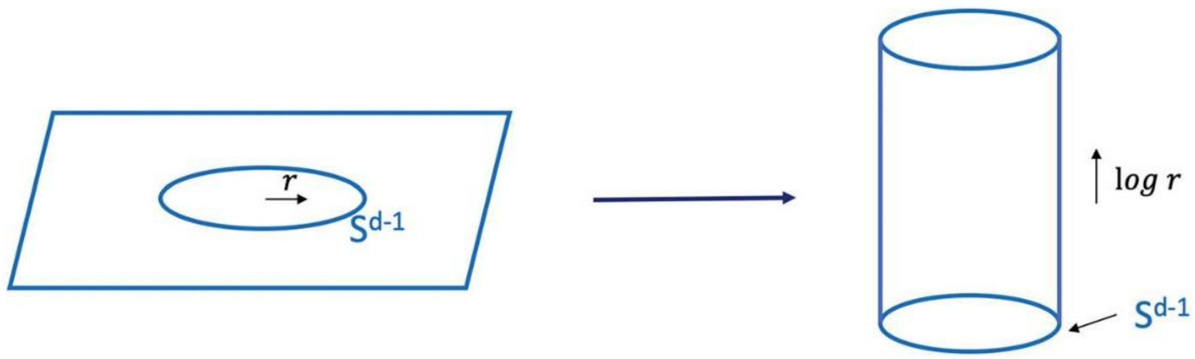


Fig. 4 On the left side, we have the Euclidean plane. On the right, we have the cylinder. An operator inserted at $r = 0$ creates a state on S^{d-1} , which can be viewed as a state of the theory on the cylinder

图 4 左侧是欧几里得平面，右侧是圆柱。插入在 $r = 0$ 处的算符会在 S^{d-1} 上产生一个态，这个态可以看作该理论在圆柱上的态

An important property of a CFT is the full set of local operators that it contains. Each local operator has a spin and a scaling dimension Δ . This set of operators are in one-to-one correspondence with the set of states on the same CFT on the cylinder (time) $\times S^{d-1}$, see Fig. 4. This arises because we can write

共形场论的一个重要性质是它包含的全部局域算符集合。每个局域算符都有自旋和标度维度 Δ 。这些算符和圆柱 (时间方向) $\times S^{d-1}$ 上同一个共形场论的态集合一一对应，参见图 4，这种对应关系可以写为

$$ds^2 = dr^2 + r^2 d\Omega_{d-1}^2 = r^2 \left[(d \log r)^2 + d\Omega_{d-1}^2 \right] \tag{25}$$

so that the plane and the cylinder are Weyl equivalent. Operators inserted at the center of the plane can be viewed as creating a state for the theory on the cylinder. The scaling transformation $r \to \lambda r$ corresponds to a Euclidean time translation on the cylinder. So that the scaling dimension of an operator $O \to \lambda^{\Delta} O$, corresponds to the energy $H_\tau = \Delta$ of the theory on the Lorentzian cylinder. Since the sphere S^{d-1} has a finite volume, the spectrum is discrete. In particular, in any CFT there is an operator of spin two and dimension $\Delta = d$, which is the stress tensor. In addition, there will be infinitely many others. This precise spectrum is a distinctive signature of the CFT.

因此平面和圆柱是外尔等价的。插入平面中心的算符可看作在圆柱上的理论中生成一个态。标度变换 $r \to \lambda r$ 对应圆柱上的欧几里得时间平移。因此算符 $O \to \lambda^{\Delta} O$ 的标度维度对应洛伦兹圆柱上理论的能量 $H_\tau = \Delta$ 。由于球面 S^{d-1} 体积有限，谱是离散的。具体而言，任意共形场论中都存在一个自旋为 2、维度为 $\Delta = d$ 的算符，也就是能量动量张量。除此之外还会存在无穷多其他算符。这套精确的谱是共形场论的独特特征。

The AdS/CFT Correspondence

AdS/CFT 对应

So far, we have separately discussed gravity in AdS_{d+1} and a CFT_d . The correspondence is the claim that these two are the same [1-3]:

到目前为止，我们已经分别讨论了 AdS_{d+1} 中的引力和一个 CFT_d 。该对应指这二者是等价的 [1-3]:

Quantum gravity with asymptotically AdS_{d+1} boundary conditions is the same as a CFT_d

满足渐近 AdS_{d+1} 边界条件的量子引力等价于一个 CFT_d

In other words, a particular well-defined quantum gravity in AdS_{d+1} leads to a well-defined CFT on its boundary. Even if we do not have an independent definition of that CFT, this is a nontrivial statement that has important implications for the quantum gravity theory. Similarly, any given CFT_d can be viewed as a possible quantum gravity theory in AdS_{d+1} , though this gravity theory might not reduce to an Einstein gravity theory that is simple to analyze.

换句话说，AdS_{d+1} 中一个定义良好的特定量子引力，会在其边界上得到一个定义良好的共形场论 (CFT)。即便我们没有该 CFT 的独立定义，这仍是一个非平凡论断，对量子引力理论有重要意义。同理，任意给定的 CFT_d 都可以被视为 AdS_{d+1} 中一个可能的量子引力理论，只不过该引力理论不一定能退化为易于分析的爱因斯坦引力理论。

We will later give some examples where we have independent definitions of the quantum gravity theory as well as the CFT.

我们后续会给出一些例子，在这些例子中我们同时拥有量子引力理论和共形场论的独立定义。

General Implications

一般推论

The correspondence can be applied for a case where the metric at the asympotically AdS boundary is nontrivial, as we discussed in section ”Some Interesting Gravity Computations.” Then the claim is that [2, 3]

正如我们在“一些有趣的引力计算”一节中讨论的，该对应可以应用于渐近 AdS 边界度规非平庸的情况，结论是 [2, 3]

$$Z_{\text{Grav}}\,[h] = Z_{CFT}\,[h] \tag{26}$$

where both sides depend on the same metric $h_{\mu\nu}$. In the right-hand side, this specifies the metric of the spacetime where the CFT lives. On the left-hand side, it specifies the metric of the asymptotic boundary of the locally asymptotically AdS space. The leading order approximation to the left-hand side is given by exponentiating (minus) the classical action of the classical solution that obeys the boundary conditions set by the metric $h_{\mu\nu}$. By this method we can compute the correlation functions of the stress tensor in terms of Feynman diagrams in AdS [3], see Fig. 5. In particular, the two-point function leads to a result that is proportional to c_{grav} (14). In other words, $c_T \propto c_{\text{grav}}$. This implies that a semiclassical gravity theory corresponds to a CFT that has a large number of fields. For example, if we had a four-dimensional space with a negative cosmological constant equal in absolute value to the dark energy in our universe, then we would need a three-dimensional CFT with 10^{120} fields. This number is the ”surprising” value of the cosmological constant in our universe.

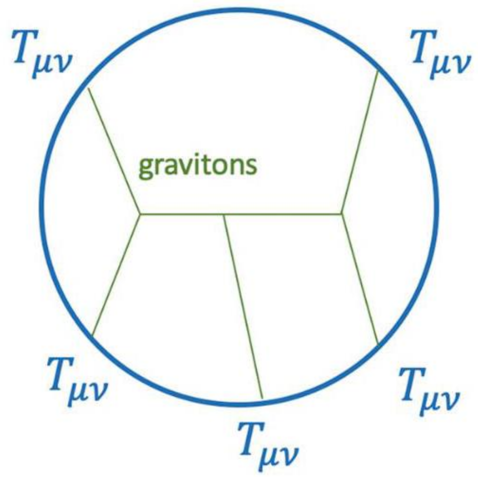
其中两侧都依赖于同一个度规 $h_{\mu\nu}$ 。在右侧，它指定了 CFT 所在时空的度规；在左侧，它指定了局部渐近 AdS 空间渐近边界的度规。左侧的领头阶近似由满足该度规 $h_{\mu\nu}$ 设定边界条件的经典解的 (负) 经典作用量取指数得到。通过这种方法，我们可以利用 AdS 中的费曼图计算能量动量张量的关联函数 [3]，参见图 5。特别地，两点函数的结果正比于 c_{grav} (14)。换言之，$c_T \propto c_{\text{grav}}$ 。这说明半经典引力理论对应于场数很多的 CFT。例如，如果我们有一个四维空间，其负宇宙学常数的绝对值等于我们宇宙中的暗能量，那么我们需要一个具有 10^{120} 个场的三维 CFT。这个数目正是我们宇宙中宇宙学常数“令人惊讶”的取值。

The correspondence also relates the spectrum of the CFT on (time) $\times S^{d-1}$ to the spectrum of the gravity theory in global AdS. In particular, the graviton is related to the stress tensor. Multigraviton states are related to products of stress tensor operators, etc.

该对应还将 (时间) $\times S^{d-1}$ 上 CFT 的谱联系到整体 AdS 中引力理论的谱。具体来说，引力子对应能量动量张量，多引力子态对应能量动量张量算符的乘积，以此类推。

Fig. 5 The computation of correlation functions on the boundary can be done in terms of bulk Feynman diagrams (Witten diagrams). Here we show a single example involving stress tensors on the boundary and gravitons in the bulk

图 5 边界上关联函数的计算可以通过体费曼图 (威顿图) 完成。这里我们展示了一个涉及边界上能量动量张量和体中引力子的简单例子



A very interesting application involves black holes. We can consider the Schwarzschild AdS black hole we discussed around (21). This is a configuration with large energy in the regime that we can trust gravity (14). Therefore, it corresponds to a large energy or large scaling dimension configuration in the CFT. More precisely, it an ensemble of states in the CFT. It could be viewed as a configuration at finite temperature, which also has a large entropy. In particular, the entropy of the black hole arises from the logarithm of the number of states in the boundary theory. So this gives a microscopic and statistical explanation for the entropy of black holes (The first microscopic computation of a black hole entropy was done for extremal black holes in [23].). In addition, the time evolution is clearly unitary in the boundary CFT. So, according to the correspondence, the black hole also evolves in a unitary fashion. This means that information is not lost in processes involving AdS black holes, in contrast with the arguments in [24]. The processes that are easy to translate between the bulk and the boundary are processes where we perturb the black hole by sending or receiving perturbations in the asymptotic boundary region. This corresponds to acting with simple local operators in the CFT. These are processes where we look at the black hole from the outside. So an important conclusion is that black holes, as seen from the outside, can be described in terms of ordinary quantum systems and evolve in a unitary fashion. The description of the interior of the black holes is more subtle and can require degrees of freedom, which are not part of the CFT as we will discuss below.

一个非常有趣的应用和黑洞有关。我们可以考虑 (21) 式附近讨论的史瓦西 AdS 黑洞，这是一个在引力可靠的区间内能量很高的构型 (14)，因此它对应 CFT 中高能量或大标度维度的构型。更准确地说，它是 CFT 中的一个态系综，可以被看作有限温度下的构型，同时也具有很大的熵。具体来说，黑洞的熵来源于边界理论中状态数的对数，这就给出了黑洞熵的微观统计解释 (第一个黑洞熵的微观计算是针对极端黑洞完成的，见 [23])。此外，边界 CFT 中的时间演化显然是幺正的，因此根据该对应，黑洞的演化也是幺正的，这说明 AdS 黑洞过程中信息不会丢失，和 [24] 中的论点不同。那些容易在体和边界之间转换的过程，是在渐近边界区域发送、接收扰动来扰动黑洞的过程，对应于在 CFT 中作用简单的局域算符，这些过程是我们从外部观测黑洞的过程。因此我们得到一个重要结论: 从外部观测的黑洞可以用普通量子系统描述，其演化是幺正的。黑洞内部的描述则更微妙，可能需要我们接下来会讨论的、不属于 CFT 的自由度。

The $AdS_5 \times S^5$ and $\mathcal{N} = 4$ Yang Mills Example

$AdS_5 \times S^5$ 与 $\mathcal{N} = 4$ 杨-米尔斯例子

Here we give an example where we specify both the gravity theory and the CFT in $d = 4$. In four dimensions, a U (N) gauge theory, a Yang-Mills theory,

我们在此给出一个例子，同时指定了 $d = 4$ 下的引力理论和共形场论。在四维中，U (N) 规范理论即杨-米尔斯理论,

$$I = -\frac{1}{4g_{YM}^2} \int \mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{\theta}{8\pi^2} \int \mathrm{Tr}\left[F \wedge F\right], \; F = dA + i\left[A, A\right] \tag{27}$$

is conformal invariant as a classical theory. Here $A = T^a A_\mu^a dx^\mu$, where T^a are $N \times N$ matrices, which generate $U(N)$. As a quantum theory, it is not conformal invariant [25]. However, if we add additional fermions and scalar fields, then it can be conformal invariant. An example is the most supersymmetric version of Yang-Mills theory [26], which has four supersymmetries, $\mathcal{N} = 4$. This theory contains (27) plus four fermions and six scalars, all in the adjoint representation of the gauge group, with interactions fixed by supersymmetry. This theory is conformal invariant as a full quantum mechanical theory [27]. It is specified by the gauge group, which we pick to be $U(N)$ and the gauge coupling g_{YM} and the θ angle, both dimensionless parameters. Since we have N colors, the effective interaction strength is proportional to

作为经典理论是共形不变的。此处 $A = T^a A_\mu^a dx^\mu$ 中 T^a 为 $N \times N$ 矩阵，生成 $U(N)$。作为量子理论，它不具有共形不变性 [25]。但如果我们额外加入费米子和标量场，它就可以具备共形不变性。一个例子是杨-米尔斯理论拥有最多超对称的版本 [26]，它具有四个超对称性，即 $\mathcal{N} = 4$。该理论包含式 (27) 外加四个费米子和六个标量场，所有场都属于规范群的伴随表示，相互作用由超对称性确定。该理论作为完整量子力学理论是共形不变的 [27]。它由规范群确定，我们取规范群为 $U(N)$，加上规范耦合 g_{YM} 和 θ 角，二者都是无量纲参数。由于我们有 N 种颜色，有效相互作用强度正比于

$$\lambda = g_{YM}^2 N \tag{28}$$

When $\lambda \ll 1$, the interactions among the gauge fields (and the matter fields) are weak. When λ is or order one or larger, the interactions are strong, and it is harder to do computations in this CFT. However, exploiting various symmetries such as supersymmetry and integrability, many computations have been done. In addition to the conformal symmetry, this theory has an $SO(6) \sim SU(4)$ symmetry that rotates the six scalars and the four fermions. Combined with supersymmetry, this forms a superconformal group that goes under the name $PSU(4 \mid 4)$.

当 $\lambda \ll 1$ 时，规范场 (和物质场) 之间的相互作用很弱。当 λ 为一阶或更大时，相互作用很强，在这个共形场论中计算难度更大。但利用超对称、可积性等多种对称性，人们已经完成了很多计算。除共形对称性外，该理论还存在一个 $SO(6) \sim SU(4)$ 对称性，可旋转六个标量和四个费米子。结合超对称性后，这形成了一个名为 $PSU(4 \mid 4)$ 的超共形群。

The corresponding AdS_5 theory is obtained from a ten-dimensional string theory on $AdS_5 \times S^5$. This ten-dimensional string theory reduces to a certain (super)gravity theory in ten dimensions at low energies. This gravity theory contains a higher form electric and magnetic field strength with five indices $F_{\mu_1 \cdots \mu_5}$. (The ordinary electromagnetic field strength has two antisymmetric indices). The flux of this magnetic flux on S^5 is quantized to an integer $N, \int_{S^5} F_5 \propto N$. This integer is identified with the number of colors, N , of the gauge theory. The value of the string coupling, g_s , is given by the expectation value of a scalar field. It turns out that $g_s \propto g_{YM}^2$. Similarly, there is a second scalar field that should be identified with the θ angle. This $AdS_5 \times S^5$ solution is invariant under the $\widetilde{SO}(2,4)$ isometries of AdS_5 , the $SO(6)$ rotation symmetry of the five-sphere and also 32 supersymmetry transformations. All of these form the supergroup $PSU(4 \mid 4)$, the same we had in the CFT. This is a first consistency check of the correspondence, both sides have the same symmetries.

对应的 AdS_5 理论得自 $AdS_5 \times S^5$ 上的十维弦理论。这个十维弦理论在低能下约化为某种十维 (超) 引力理论。该引力理论包含带五个指标 $F_{\mu_1 \cdots \mu_5}$ 的高阶形式电磁通量。(普通电磁场强度拥有两个反对称指标)。S^5 上该磁通量的通量被量子化为整数 $N, \int_{S^5} F_5 \propto N$。这个整数对应规范理论的颜色数 N。弦耦合常数 g_s 由一个标量场的期望值给出，最终得到 $g_s \propto g_{YM}^2$。类似地，存在第二个标量场对应 θ 角。这个 $AdS_5 \times S^5$ 解在 AdS_5 的 $\widetilde{SO}(2,4)$ 等距群、五球面的 $SO(6)$ 旋转对称性以及 32 个超对称变换下保持不变。所有这些对称性共同构成了超群 $PSU(4 \mid 4)$，和共形场论中的超群一致。这是对 AdS/CFT 对应性的第一个一致性检验: 两侧具有相同的对称性。

The ten-dimensional Einstein equations then imply that the radius of curvature of both the AdS_5 and the S^5 scales like $R \propto N^{1/4} l_p$, in terms of the ten-dimensional Planck scale. This means that N needs to be large, $N \gg 1$, in order for us to trust the gravity approximation, and also for strings to be weakly coupled.

十维爱因斯坦方程表明，以十维普朗克尺度衡量，AdS_5 和 S^5 的曲率半径都按 $R \propto N^{1/4} l_p$ 标度。这意味着 N 需要很大，即满足 $N \gg 1$ ，我们才能相信引力近似，同时保证弦处于弱耦合。

In string theory, there is a further requirement for gravity to be a good approximation, which is that the radius of curvature should be larger than the string length. The radius of curvature in units of the string length is

在弦论中，引力要成为良好近似还存在另一项要求，即曲率半径应当大于弦长。以弦长为单位的曲率半径是

$$\frac{R^4}{l_s^4} = g_{YM}^2 N = \lambda \tag{29}$$

Therefore, to trust the gravity solution, we need $\lambda \gg 1$. This is perfectly complementary to the regime $\lambda \ll 1$, which is when the gauge theory is weakly coupled. This implies that when one side is easy to analyze, the other is hard, and vice versa. This makes the conjecture hard to test, but it also makes it useful if true, since it transforms a difficult problem into an easy one.

因此，要让引力解成立，我们需要 $\lambda \gg 1$ 。这与规范理论弱耦合的区间 $\lambda \ll 1$ 完全互补。这意味着当其中一方易于分析时，另一方就难以分析，反之亦然。这一点让该猜想很难被检验，但如果猜想成立，它也因此十分有用——因为它能将难题转化为简单问题。

There is an interesting limit that consists of taking $N \to \infty$ keeping λ fixed. In this limit, a simple argument suggests that the theory should become some kind of string theory [28]. The idea is that only certain planar Feynman diagrams contribute in this limit. In other words, particles form chains according to the way the color indices are contracted, and different chains interact weakly with each other. It was initially thought that these strings would live in four dimensions, but it was suggested in [29] that they could live in five dimensions. Here we see they live in ten dimensions.

存在一个有趣的极限: 取 $N \to \infty$ 同时保持 λ 固定。在该极限下，一个简单的推导表明该理论应当会成为某种弦论 [28]。其核心观点是，该极限下只有特定的平面费曼图有贡献。换句话说，粒子会根据色指标缩并的方式形成链，不同链之间的相互作用很弱。最初人们认为这些弦存在于四维空间中，但文献 [29] 提出它们可能存在于五维空间中。而我们此处看到它们存在于十维空间中。

These planar diagrams lead to an integrable system for this particular theory [30]. This integrable structure is also present for quantum strings moving in $AdS_5 \times S^5$. The energies of these states can be computed exactly as function of λ , interpolating continuously between the weak coupling $\lambda \ll 1$ and strong coupling $\lambda \gg 1$ results. This shows that the chain of gluons that we have at weak coupling metamorphoses into the smooth ten-dimensional string we have at strong coupling.

这些平面图为该特殊理论给出了一个可积系统 [30]。这种可积结构同样存在于在 $AdS_5 \times S^5$ 中运动的量子弦上。这些态的能量可以作为 λ 的函数被精确计算，在弱耦合 $\lambda \ll 1$ 结果和强耦合 $\lambda \gg 1$ 结果之间连续插值。这表明我们在弱耦合下得到的胶子链会演变为强耦合下平滑的十维弦。

Just to be more concrete on this point. At large N , the spectrum can be described in terms of a Fock space. This Fock space is generated by single trace operators in the gauge theory and single string states in the bulk. These single trace operators have the schematic form

为了把这一点讲得更具体。在大 N 下，能谱可以用福克空间描述。这个福克空间由规范理论中的单迹算符和体中的单弦态生成。这些单迹算符的概型形式为

$$\mathrm{Tr}\left[\Phi_1\Phi_2\cdots\Phi_n\right] \tag{30}$$

where Φ denotes a gauge field strength $F_{\mu\nu}$, a fermion field $\psi_{\alpha,a}$ or scalar ϕ^i , or any number of derivatives acting on these fields, such as $D_{\mu_1}D_{\mu_2}\phi^i$ for example. These fields are in the adjoint representation so we can think of them as $N \times N$ matrices in color space so that in (30) we are taking the trace of the product of these matrices. There is a large number of operators of this kind that we can write. At zero coupling, their conformal dimension is just the sum of the conformal dimensions of each of the "letters" Φ_i that make the operator. At strong coupling, most of these fields acquire a large conformal dimension. There is only a small fraction of these that retain their zero coupling dimension. These are operators that are in small or special representations of $PSU(4\mid 4)$, which are called BPS representations. On the gravity side, these are all the ten-dimensional massless fields of the supergravity theory. All other operators become massive string states at strong coupling. The lowest massive string state has a mass of order $1/l_s$ in ten dimensions and leads to an anomalous dimension of order $\Delta \sim R/l_s \propto \lambda^{1/4}$ for $\lambda \gg 1$. At weak coupling the lightest non-BPS operator is the operator $\mathrm{Tr}\left[\phi^i\phi^i\right]$ which has dimension $\Delta = 2$ when $\lambda = 0$. Its dimension can be computed for any value of λ using the methods of integrability, which gives an interpolating function that goes between $\Delta = 2$ for zero λ and $\Delta \sim 2\lambda^{1/4}$ for large $\lambda \gg 1$ [30]. This constitutes a very strong test of the duality in the large N limit.

其中 Φ 表示规范场强度 $F_{\mu\nu}$、费米子场 $\psi_{\alpha,a}$ 或标量场 ϕ^i ，也可以是作用在这些场上任意阶导数，例如 $D_{\mu_1}D_{\mu_2}\phi^i$ 。这些场属于伴随表示，因此我们可以将它们视为色空间中的 $N\times N$ 矩阵，这样 (30) 式就是对这些矩阵的乘积求迹。我们可以写出大量这类算符。在零耦合时，它们的共形维度就是构成算符的每个“单元” Φ_i 的共形维度之和。在强耦合下，大部分这类场都会获得很大的共形维度。只有很少一部分能保留零耦合时的共形维度。这些算符属于 $PSU(4\mid 4)$ 的小表示或特殊表示，被称为 BPS 表示。在引力侧，这些对应超引力理论中所有十维无质量场。所有其他算符在强耦合下都会成为有质量弦态。最低的有质量弦态在十维中的质量量级为 $1/l_s$, 对 $\lambda \gg 1$ 给出量级为 $\Delta \sim R/l_s \propto \lambda^{1/4}$ 的反常维度。在弱耦合下，最轻的非 BPS 算符是算符 $\mathrm{Tr}\left[\phi^i\phi^i\right]$，当 $\lambda = 0$ 时它的维度为 $\Delta = 2$ 。利用可积性方法可以对任意 λ 的值计算它的维度，得到一个插值函数，在零 λ 时为 $\Delta = 2$ ，大 $\lambda \gg 1$ 时为 $\Delta \sim 2\lambda^{1/4}$ [30]。这是对大 N 极限下该对偶的非常有力的检验。

One important property of the spectrum of single particle states is the following. At weak coupling there are many light states with spins $S > 2$. This reflects the fact that different particles making up the chain (30) move independently on the S^3 . However, at strong coupling, any state with spin $S > 2$ has an anomalous dimension of order $\lambda^{1/4}$. So at strong coupling only very special collective excitations remain light. We expect that any theory with weakly coupled higher spin particles that includes the graviton would reduce to Einstein gravity if $\Delta_{S>2} \geq \Delta_{\text{gap}}$, with $\Delta_{\text{gap}} \gg 1$. Then gravity approximation will be good for distances larger than R/Δ_{gap} [31] .

单粒子态能谱的一个重要性质如下: 在弱耦合下，存在大量自旋为 $S > 2$ 的轻态。这反映了构成链 (30) 的不同粒子在 S^3 上独立运动。但在强耦合下，任何自旋为 $S > 2$ 的态都具有量级为 $\lambda^{1/4}$ 的反常维度。因此强耦合下只有极少数特殊的集体激发保持为轻态。我们认为，若 $\Delta_{S>2} \geq \Delta_{\text{gap}}$ 且 $\Delta_{\text{gap}} \gg 1$，任何包含引力子、具有弱耦合高自旋粒子的理论都会约化为爱因斯坦引力，此时引力近似对大于 R/Δ_{gap} [31] 的距离适用。

Note that the bulk string theory has a well-defined perturbative expansion, without unknown counterterms (as opposed to the case of gravity as an effective theory). So, in principle, we can check it to any order in the $g_s \sim 1/N$ expansion. Furthermore, in some cases the exact answer is known, including all non-perturbative terms [32, 33] .

需要注意的是，体弦理论具有定义良好的微扰展开，不存在未知抵消项 (这与作为有效理论的引力不同)。因此原则上，我们可以在 $g_s \sim 1/N$ 展开中任意阶验证它。此外，在某些情况下我们已经得到了包含所有非微扰项 [32, 33] 的精确解。

There are other AdS/CFT pairs where we know both the CFT and the bulk theory. Another highly symmetric example involves AdS_4 . The dual theory is a Chern Simons matter theory, a theory of massless interacting non-abelian anyons [34]. In this case, the bulk theory could be viewed as a ten-dimensional theory on $AdS_4 \times CP^3$. Or, for a special value of the parameters, as an 11-dimensional theory on $AdS_4 \times S^7$. The partition function on the S^3 can be computed on the CFT and matched to the expression obtained in gravity as in (20) [35,36].

还有其他我们既已知 CFT 也已知体理论的 AdS/CFT 对。另一个高度对称的例子涉及 AdS_4 。其对偶理论是陈-西蒙斯物质理论，这是一种无质量相互作用非阿贝尔任意子理论 [34]。在该情形下，体理论可以看作是 $AdS_4 \times CP^3$ 上的十维理论；对于参数的特殊取值，它也可以是 $AdS_4 \times S^7$ 上的十一维理论。我们可以在 CFT 中计算 S^3 上的配分函数，并与引力中如 (20) 得到的表达式匹配 [35,36]。

In all the known examples where we know both the CFT and the bulk theory, the bulk theory is a 10- or 11-dimensional theory on a space with an AdS factor (or warped factor). We do not have a general method for generating dual pairs, namely to find the CFT if the bulk is known, or vice versa. We will recall in the next subsection the particular setup that gave rise to the original examples. From these, we can generate others by turning on marginal deformations. Alternatively, we can turn on relevant deformations that lead to a renormalization group flow to a new fixed point.

在所有我们既已知 CFT 也已知体理论的现有例子中，体理论都是带有 AdS 因子 (或弯曲因子) 的空间上的十维或十一维理论。我们目前没有生成对偶对的通用方法，即已知体理论找 CFT，或是反过来。我们将在下一小节回顾产生最初例子的特定构造。在此基础上，我们可以通过开启边缘形变生成其他对偶对；也可以开启相关形变，引发重整化群流流向新的不动点。

Why Is the Correspondence Reasonable?

为什么这个对应是合理的?

The original example arose by thinking about black holes and black branes in string theory. The discovery of D-branes [37] provided the essential link. D-branes are relatively heavy objects, which are, in some sense, between elementary particles and black holes. More precisely, they are a type of soliton in string theory whose excitations have a very precise mathematical description involving open strings. The low energy limit of this description reduces to supersymmetric versions of gauge theories. The transverse positions of D-branes become non-abelian scalar fields. For example, in the case of 3-branes in ten dimensions, this low-energy theory is precisely $\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory. At the same time, D-branes carry the generalized electric and magnetic fluxes of the supergravity theories. In particular, they source the five form field strength with five indices F_5 . If this flux is large, the backreaction is large, and we get an extremal black brane solution [38]. This solution has a region near the horizon with the $AdS_5 \times S_5$ geometry. This region has a relatively small value of the redshift factor $g_{00} \ll 1$, due to the geometry of AdS_5 . This means that general excitations living in the near horizon geometry appear to have very low energy as seen from the outside. So it is tempting to identify the two low-energy descriptions of D-branes: the gauge theory with the $AdS_5 \times S_5$ region of the geometry [1].

最初的例子源自对弦论中黑洞与黑膜的思考。D 膜的发现 [37] 提供了关键连接。D 膜是质量相对较大的物体，从某种意义上说，它介于基本粒子与黑洞之间。更准确地说，它是弦论中的一类孤子，其激发态有着由开弦描述的非常精确的数学形式。该描述的低能极限退化为超对称规范场论。D 膜的横向位置变为非阿贝尔标量场。例如，对于十维空间中的 3 膜，这个低能理论恰好就是 $\mathcal{N} = 4$ 超对称 $U(N)$ 规范理论。同时，D 膜携带超引力理论的广义电通量和磁通量，具体来说，D 膜是带五个指标的五形式场强 F_5 的源。如果该通量很大，背景反作用就很强，我们就会得到一个极端黑膜解 [38]。这个解的视界附近区域存在 $AdS_5 \times S_5$ 几何。受 AdS_5 几何的影响，该区域的红移因子 $g_{00} \ll 1$ 值相对较小。这意味着从外部观测，生活在近视界几何中的普通激发都显得能量极低。因此我们很自然地将 D 膜的两种低能描述等同起来: 规范理论对应几何的 $AdS_5 \times S_5$ 区域 [1]。

There are other roads that might also have taken to the same answer. We could start from $\mathcal{N} = 4$ super Yang-Mills at large N . Then one can argue that gauge theories at large N give rise to strings. Due to a conformal anomaly on the string worldsheet, such a string should move in more than four dimensions [29]. In order to realize the conformal symmetry of the gauge theory, one could guess that the geometry should contain an AdS_5 factor. Having added one extra dimension, we could add five more and get to ten dimensions where we have well-understood string theories. This leads us to consider $AdS_5 \times S^5$, which has the same $PSU(4 \mid 4)$ symmetries as the gauge theory.

还有其他思路也能得到相同结论。我们可以从大 N 下的 $\mathcal{N} = 4$ 超 Yang-Mills 理论出发。可以证明，大 N 下的规范理论会产生弦。由于弦世界面上存在共形反常，这种弦应当在大于四维的空间中运动 [29]。为了实现规范理论的共形对称性，我们可以推测该几何应当包含一个 AdS_5 因子。增加一个额外维度后，我们可以再增加五个维度，最终得到十维空间，而我们对十维弦论已经有充分的理解。这就引导我们考虑 $AdS_5 \times S^5$ ，它和规范理论拥有相同的 $PSU(4 \mid 4)$ 对称性。

Another possible road one might have taken involves ideas motivated by the Bekenstein black hole entropy formula [39]. Since the entropy is given by the area rather than the volume one expects that in a quantum theory of gravity, there should be a sort of dimensional reduction [40, 41] analogous to the way that an optical hologram stores a three-dimensional image on a two-dimensional surface. Anti-de-Sitter space is a natural case where there is a boundary far away where this dimensional reduction could take place [3]. There the symmetries suggest a correspondence to a CFT. Starting from $AdS_5 \times S_5$ and looking for a CFT with the right symmetries, one would have been led to guess a correspondence to $\mathcal{N} = 4$ super Yang Mills.

另一种可能的思路来自贝肯斯坦黑洞熵公式启发的思想 [39]。由于黑洞熵由面积而非体积给出，我们可以预期，在量子引力理论中应当存在某种维度约化 [40, 41]，就像光学全息图在二维表面上存储三维图像一样。反德西特空间是这类维度约化自然发生的场景，它拥有一个遥远的边界 [3]。这里的对称性暗示了它与 CFT 的对应。从 $AdS_5 \times S_5$ 出发，寻找具有对应对称性的 CFT，我们就会得到它与 $\mathcal{N} = 4$ 超 Yang-Mills 的对应猜想。

The original argument used D-branes as a crucial bridge to propose the duality. By considering branes at geometric singularities, it is also possible to generate many other examples. By now there are infinite families of dual pairs, for which we have various amounts of evidence. All of these dual pairs involve a 10- or 11-dimensional background of string or M-theory.

最初的论证以 D 膜作为关键桥梁提出了这一对偶性。通过考虑几何奇点处的膜，我们还可以得到许多其他例子。到目前为止，已经存在无穷多组对偶对，我们也收集到了不同程度的证据。所有这些对偶对都涉及弦论或 M 理论的 10 维或 11 维背景。

However, we do not know whether there are other examples. One difficulty is that it is hard to construct conformal field theories. It is also hard to construct consistent quantum theories of gravity, all the examples that we have come from compactifications of 10-dimensional string theory or its 11 dimensional M-theory cousin.

但我们目前还不清楚是否存在其他例子。一个难点在于共形场论本身很难构造，自洽的量子引力理论也同样难以构造，我们现有的所有例子都来自 10 维弦论或其 11 维的 M 理论推广的紧致化。

There are some interesting large N CFTs. For example, $\mathcal{N} = 1$ SUSY QCD with an $\frac{3N_c}{2} < N_f < 3N_c$ is conformal in the IR [42]. For large N one might expect these to have a weakly coupled limit. However, the large flavor symmetry seems to be problematic for having a weakly coupled dual limit, and we do not know of a good dual candidate. Similar fixed points are expected to exist in non-supersymmetric QCD.

存在一些有趣的大 N 共形场论。例如，带有 $\frac{3N_c}{2} < N_f < 3N_c$ 的 $\mathcal{N} = 1$ 超对称量子色动力学在红外区是共形的 [42]。对于大 N，我们或许可以期待它们存在弱耦合极限。然而，大味对称性对于存在弱耦合对偶极限而言存在问题，目前我们尚未找到合适的对偶候选。非超对称量子色动力学中被认为也存在类似的不动点。

One interesting class of conformal field theories are $d = 3O(N)$ Wilson-Fischer fixed points. These are not supersymmetric. It has been proposed in [43] that these theories are dual to a gravity theory (Vasiliev-gravity [44]) with an infinite number of almost massless fields with spins $S > 2$. In particular, these large N theories are not dual to an Einstein gravity theory. The weak coupling limit, $\lambda \ll 1$, of $\mathcal{N} = 4$ super Yang-Mills also involves an infinite number of nearly massless higher spin fields in the bulk. These examples highlight the fact that the general dual of a CFT might not be a standard Einstein gravity theory. Einstein gravity theories seem to arise only in some cases, but we do not know how representative those cases are within the set of possible CFTs.

一类有趣的共形场论是 $d = 3O(N)$ 威尔逊-费舍尔不动点，它们是非超对称的。文献 [43] 提出，这些理论对偶于一个引力理论 (瓦西里耶夫引力 [44])，该引力理论拥有无穷多个自旋为 $S > 2$ 的近无质量场。具体而言，这些大 N 理论并不对偶于爱因斯坦引力理论。$\mathcal{N} = 4$ 超杨-米尔斯的弱耦合极限 $\lambda \ll 1$ 在体空间中也包含无穷多个近无质量的高自旋场。这些例子突显了一个事实: 共形场论的一般对偶不一定是标准的爱因斯坦引力理论。爱因斯坦引力理论似乎仅在部分情形下出现，我们目前并不清楚这些情形在所有可能的共形场论中的代表性如何。

Entanglement and Geometry

纠缠与几何

An interesting geometry to consider is the double-sided Schwarzschild black hole. This is simply the full analytic extension of the Lorentzian version of (21). Its Penrose diagram can be seen in Fig. 6. This geometry contains two asymptotic boundaries. These two boundaries are disconnected on the boundary, but they are connected through the bulk. The bulk geometry contains horizons, and it is not possible to send a signal from one boundary to the other. The idea is that we have two separate CFTs, each describing one of the boundaries. The geometry is dual to an entangled state on the product of the Hilbert spaces describing the left CFT and the right CFT [51]. For this geometry, it is a special entangled state called the ”thermofield double”

值得研究的一个有趣几何是双侧史瓦西黑洞，它就是式 (21) 洛伦兹版本的完整解析延拓。其彭罗斯图可见图 6。该几何包含两个渐近边界，两个边界在边界上不连通，但通过体相连。体几何包含视界，因此无法从一个边界向另一个边界发送信号。一般的理解是，我们有两个独立的共形场论 (CFT)，每个描述一个边界。该几何对偶于左 CFT 和右 CFT 希尔伯特空间乘积上的一个纠缠态 [51]。对该几何来说，它是一个特殊的纠缠态，称为“热场双态”

$$|TFD\rangle \propto \sum_n e^{-\beta E_n/2}|\bar{E}_n\rangle_L|E_n\rangle_R \tag{31}$$

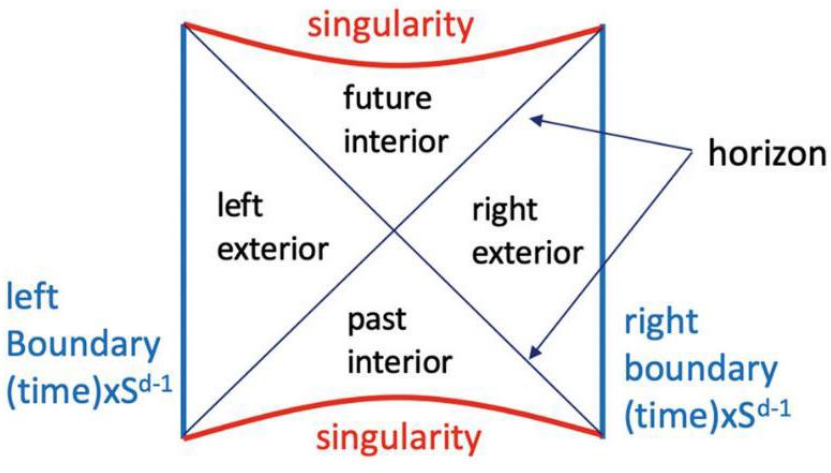


Fig. 6 The two-sided Schwarzschild AdS black hole Penrose diagram. On each point of this diagram we have an S^{d-1} . We have two disconnected cylindrical boundaries. They are connected in the interior. Since

light rays propagate at 45 degrees we see that is not possible to send a signal from the left boundary to the right boundary

图 6 双侧史瓦西 AdS 黑洞彭罗斯图。该图的每个点上都有一个 S^{d-1} 。我们有两个不连通的柱形边界，它们在内部相连。由于光线沿 45 度传播，可见无法从左边界向右边界发送信号

where the sum runs over all the energy eigenstates of a single CFT on (time) $\times S^{d-1}$. Since we cannot send information using entanglement, this is consistent with the fact that the horizons prevent communication between the two sides. Furthermore, any deformations of the bulk state also lead to geometries for which the two boundaries are disconnected. This is a general property of Lorentzian geometries with multiple boundaries [48]. Then, general multiboundary geometries can be interpreted as entangled states, and interesting ones can be produced by suitable perturbations of the two-sided black hole [49]. It is also possible to introduce a coupling between the two sides, which can make the wormhole traversable [50] in a way that is connected with quantum teleportation. One can speculate that for general systems, there is some form of geometric connection between entangled states, an idea that is described by the ER=EPR slogan [56].

其中求和遍历单个 CFT 在 (时间) $\times S^{d-1}$ 上的所有能量本征态。由于我们无法利用纠缠传递信息，这与视界阻止两侧通信的结论一致。此外，体态的任何形变也会得到两侧边界不连通的几何。这是多边界洛伦兹几何的一般性质 [48]。因此，一般的多边界几何可以解释为纠缠态，通过对双侧黑洞做适当扰动可以构造出许多有趣的情形 [49]。我们还可以在两侧之间引入耦合，这能让虫洞可穿行 [50]，该过程与量子隐形传态有关。我们可以猜想，对一般系统而言，纠缠态之间存在某种形式的几何连接，这个想法被概括为 ER=EPR 口号 [56]。

In these cases, one can wonder how much of the bulk is described by each CFT on its own. The answer to this question is suggested by a very important development for the study of the correspondence, which is a gravitational formula that measures the von-Neumann (or fine grained) entropy of the bulk state [52-55]. This formula is somewhat similar to the black hole entropy formula since it gives the entropy in terms of the area of a surface. The difference lies in how we pick the surface. The formula is

在这些情形下，我们可以追问单个 CFT 自身能描述多大的体区域。这个问题的答案来自对应关系研究中一项非常重要的进展: 人们得到了一个引力公式，用来度量体态的冯·诺依曼 (即细粒度) 熵 [52-55]。该公式和黑洞熵公式有些类似，因为它都将熵表示为某个曲面的面积，区别在于选取曲面的规则不同。公式如下

$$S = \min\left\{\text{ext}\left[\frac{A_X}{4G_N} + S_{\sum_X}\right]\right\} \tag{32}$$

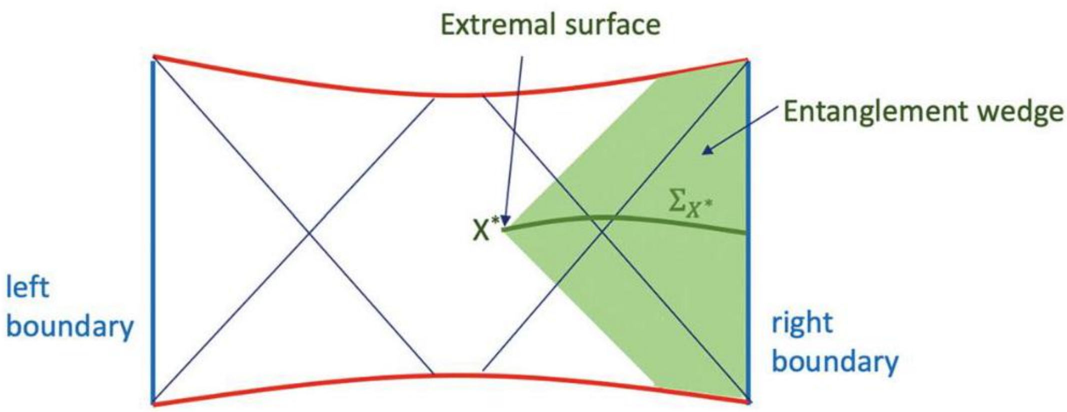


Fig. 7 We consider a wormhole geometry with matter inside so that it is longer than the one in Fig. 6. We consider the entanglement wedge of the right side. The location of X^* depends on the details of the geometry, but it is behind the horizon for the right side. The Entanglement wedge is the full domain of dependence of the surface $\sum_X$

图 7 我们考虑一个内部存在物质的虫洞几何，它比图 6 中的虫洞更长。我们考察右侧的纠缠楔，X^* 的位置取决于几何的细节，但它位于右侧视界的后方。纠缠楔是曲面 $\sum_X$ 的整个依赖域

We first compute the quantity in the brackets where X is a codimension two surface, A_X is its area, and $\sum_X$ is a codimension one space-like surface that goes from X to the boundary. $S_{\sum_X}$ is the entropy of the quantum fields of the semiclassical gravity description of the state. Then we are supposed to extremize with respect to the choice of X . If there is more than one extremum, we are supposed to consider the one with the minimal value. This formula suggests an answer for the region that is described by a single boundary CFT: it is the region that is uniquely determined by initial conditions on the surface $\sum_{X^*}$ where X^* is the surface we find after the extremization and minimization process. By ”described” we mean that we can recover the quantum state of a probe qubit living in this region by performing operations purely on the single boundary CFT. This is called ”entanglement wedge reconstruction hypothesis” [57]. For a review of recent applications to the black hole information problem, see [58].

我们首先计算括号中的量: 其中 X 是余维数为 2 的曲面，A_X 是它的面积，$\sum_X$ 是从 X 延伸到边界的余维数为 1 的类空曲面，$S_{\sum_X}$ 是该态半经典引力描述中量子场的熵。接下来我们需要对 X 的选取做极值化，如果存在多个极值，我们选取取值最小的那个。该公式对单个边界 CFT 能描述的区域给出了一个答案: 该区域由曲面 $\sum_{X^*}$ 上的初值条件唯一确定，其中 X^* 是我们经过极值化和最小化过程后得到的曲面。此处“描述”的意思是，我们可以通过仅在单个边界 CFT 上操作，重构出该区域内探测量子比特的量子态。这就是“纠缠楔重构假设”[57]。有关该假设在黑洞信息问题中近期应用的综述，参见 [58]。

A related set of interesting developments that we will not discuss are the qualitative connection between ”tensor networks” and the structure of the bulk [59] as well as connections between the extraction of bulk information and quantum error correction [60] (Fig. 7).

另一相关的有趣进展我们不在此讨论，包括“张量网络”和体结构之间的定性联系 [59]，以及提取体信息和量子纠错之间的关联 [60](图 7)。

Beyond Anti-de-Sitter Space

超越反德西特空间

Nonconformal Theories

非共形理论

Once the correspondence is true for a CFT, then it is clear that we can add a single trace relevant deformation. This corresponds to changing the boundary value for a scalar field at the AdS boundary. Then this field has a nonzero vev in the bulk that breaks the scaling symmetry. However, we expect that the two sides should still be dual to each other. We can generate such flows by adding masses to the $\mathcal{N} = 4$ theory, see e.g.,[46]. A theory that is nearly conformal is the SYK model [45], which has many similarities with near extremal black holes and involves a spacetime that is nearly AdS_2 . In this case, we do not have a clear bulk definition of the dual gravity theory, and it does not appear to have a local bulk Lagrangian. It is also possible to generate nonconformal examples starting from other D- p -branes, with $p < 3$ [47].

一旦对应关系对共形场论成立，显然我们就可以添加单迹 Relevant 形变。这对应改变标量场在 AdS 边界处的边界值。之后该场在体空间获得非零真空期望值，破坏了标度对称性。不过我们仍预期这两边依旧互为对偶。我们可以通过给 $\mathcal{N} = 4$ 理论添加质量来生成这类流，例子见文献 [46]。SYK 模型是近似共形的理论 [45]，它与近极端黑洞有诸多相似性，其涉及的时空近似为 AdS_2 。在该情形下，我们尚未得到对偶引力理论清晰的体空间定义，它似乎也不存在局域体空间拉格朗日量。从其他 D- p 膜出发也可以构造非共形的例子，见 $p < 3$ [47]。

Speculations About Gravity Duals for Any Quantum System

任意量子系统引力对偶的猜想

These examples suggest that any quantum system could be somehow dual to a quantum theory of gravity. So far, we only understand special cases. We do not have a general enough definition of ”quantum gravity” where this is true. It would be interesting to understand whether this is true or not. If it is not, where lies the demarcation line between systems that have a dual and those that do not? One possibility is that the bulk description only emerges as an approximation. This would make gravity somehow ”less fundamental” than the boundary theory. Perhaps the inverse is true, there is a fundamental theory of quantum gravity, and it is only in situations where we have a boundary that a quantum mechanics theory emerges.

这些例子表明，任意量子系统都可能在某种意义上对偶于一个量子引力理论。到目前为止，我们仅理解这类情况的特殊案例。我们还没有给出足够普适的“量子引力”定义，能让这一猜想成立。厘清它是否正确会是很有意义的研究。如果它不成立，那存在对偶的系统和不存在对偶的系统之间的分界线在哪里？一种可能性是，块体描述只是作为近似涌现出来的，这会让引力在某种意义上比边界理论“更不基础”。或许反过来才对: 存在一个基础的量子引力理论，量子力学理论只有在存在边界的情境下才会涌现。

Spacetimes with Other Asymptotic Structures

具有其他渐近结构的时空

There is an interesting proposal for computing the flat space S-matrix in 11 dimensions by using a quantum mechanical matrix theory [61] which is in the spirit of the AdS/CFT correspondence (and preceded it!) since it relates a quantum system to a theory of gravity.

有一个有趣的方案, 利用量子力学矩阵理论 [61] 计算 11 维平坦空间的 S 矩阵, 该方案遵循 AdS/CFT 对应思想 (且问世更早！)，因为它将一个量子系统与引力理论关联起来。

In the case of de-Sitter space, the boundary is space-like, and it lives in the far future and/or the far past, so it is natural to conjecture that it is described by a Euclidean field theory [62, 63] living there. Unfortunately, we have no example where the bulk theory is an Einstein theory of gravity.

对于德西特空间，其边界为类空边界，位于远未来和/或远过去，因此很自然可以猜想它由存在于该处的欧几里得场论描述 [62, 63]。遗憾的是，目前我们没有体理论为爱因斯坦引力理论的实例。

A Challenge for the Correspondence

对 AdS/CFT 对应性的一项挑战

There is an interesting two-dimensional theory of gravity, called *JT* gravity, that has a well-defined expansion in terms of a sum over geometries. Nevertheless, it is not dual to a quantum mechanics theory at the boundary. Instead, it is dual to an ensemble of quantum mechanical theories, a theory where the Hamiltonian is a random variable [19].

存在一个有趣的二维引力理论，名为 *JT* 引力，该理论可按几何求和进行定义明确的展开。但它并不与边界处的一个量子力学理论对偶；相反，它对偶于量子力学理论的系综，即哈密顿量为随机变量的理论 [19]。

This seems to say that we need to generalize our concept of the correspondence to consider the possibility of having random Hamiltonians, at least for examples in two dimensions or less. On the other hand, for higher dimensional theories, it seems that CFTs are rather constrained structures and that we cannot pick one at random with some natural measure. The proper way to resolve this ”tension” is still a matter of ongoing research.

这似乎表明我们需要推广我们对对应性的认知，以接受存在随机哈密顿量的可能性，至少对于二维及更低维度的例子是如此。另一方面，对于更高维的理论，共形场论 (CFT) 本身是约束较强的结构，我们无法通过某种自然测度随机选取一个共形场论。解决这一“矛盾”的正确方法目前仍是 ongoing 研究课题。

Acknowledgments This field was developed by numerous authors over many years, and we could not do justice to all their contributions. There exist other reviews and books that cover the subject more extensively

and include applications to other fields beyond problems in quantum gravity.

致谢该领域由众多学者历经多年发展而成，我们无法在此尽数公平列举所有贡献。已有其他综述与著作更全面地涵盖了本主题，且包含了其在量子引力问题之外其他领域的应用。

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